Physics IV
ISI B.Math
Midterm Exam : February 25,2019

Total Marks: 80
Time : 3 hours
Answer all questions

In all of the questions below, unless specified otherwise S' refers to an inertial frame moving with a velocity $\mathbf{v} = v\hat{\mathbf{x}}$ with respect to S.

1.(**Marks: 2 + 4 + 6 + 4 = 16**)

(a) State the two fundamental postulates of the Special Theory of Relativity.

(b) The time interval and spatial separation between two events A and B are measured to be Δt_{AB} and Δx_{AB} respectively in the frame S. What are the conditions on Δt_{AB} and Δx_{AB} such that event A is seen to occur before event B in any inertial frame S'?

(c) Set up the coordinate systems (x, ct) in S and (x', ct') in S'. On this spacetime diagram, show that i) two events A and B that are simultaneous in S are not simultaneous in S'. ii) two events P and Q that are simultaneous in S' are not simultaneous in S. Draw separate diagrams for cases i) and ii) for the sake of clarity.

(d) A bomb blast occurs in Bangalore at 7 am and in Kolkata at 12 pm. The perpetrator absolves himself of the crime by finding an inertial frame where both blasts occur simultaneously and claiming that he could not have been at two different places at the same time. Explain whether his argument can be justified.

2. (**Marks: 4 + 4 + 4 + 4 + 4 = 20**)

(a) Show that the sum of any two orthogonal spacelike four vectors is spacelike.

(b) Show that a timelike four vector and a null four vector cannot be orthogonal

(c) It is found that the x component of momentum p_x vanishes in every inertial frame. Argue that this implies that the four momentum vector must have the form $P^\mu = (0, 0, 0, 0)$ in every frame.

(d) If P^μ is the four-momentum of a particle and F^μ is the four-force defined as the product of the mass and the four-acceleration , show that $P^\mu F_\mu = 0$.

(e) Using the result in (c) , show that the four force F can be written as $F = (\frac{\gamma}{c}\mathbf{f}\cdot\mathbf{v}, \gamma\mathbf{f})$ where $\mathbf{v}$ is the three-velocity of the particle and $\mathbf{f}$, the three force $= \frac{d\mathbf{p}}{dt}(\mathbf{p} = m\gamma\mathbf{v}$. $\mathbf{p}$ being the relativistic three momentum).

3. (**Marks: 6 + 6 + 4 = 16**)

A stick of proper length l_0 lies at rest in the frame S making an angle $\theta = \tan^{-1}(\frac{3}{4})$ with the x axis . A frame S' moves with velocity $\mathbf{v} = v\hat{\mathbf{x}}$ with respect to S. In S' the stick is at an angle 45° with the x' axis.

(a) What is v ?

(b) What is the length l' of the stick measured in S' ?

(c) If there are two clocks attached to the two ends of the stick and the clocks are synchronized in the S frame and the rear clock is synchronized with the reading of the S frame, will the clocks continue to remain synchronized in the S' frame ? If not, which clock will read ahead in time, and what will be the difference between the two readings ?

4. (**Marks: 6 + 4 + 4 = 14**)

(a) A photon with frequency ν in frame S is travelling in the x direction. What is its frequency ν' measured in frame S' ?(Hint: Consider how energy transforms under Lorentz transformations)

(b) Given an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$, we have shown in class that $E^2 - c^2B^2$ and $\mathbf{E} \cdot \mathbf{B}$ remain invariant under Lorentz transformations. Show that a pure electric field in one inertial frame cannot be transformed into a pure magnetic field in another inertial frame and vice versa.

(c) In the lab frame, an object moves with a velocity $(u_x, u_y, 0)$ and you move with a velocity v in the x direction. What should v be so that you also see the object move with velocity u_y in your y direction ?

5. (**Marks: 7 + 7 = 14**)

(a) A particle with mass M and energy E decays into two identical particles. In the lab frame , one of them is emitted at an angle of 90 deg to the direction of motion of the particle of mass M. What are the energies of the created particles ?

(b) Two photons each have energy E. They collide at an angle θ and create a particle of mass M. What is M ?